

P2

Functions

In general $fg(x) \neq gf(x)$

The graph of $y = f(x)^{-1}$ is a reflection of $y = f(x)$ in the line $y = x$

Modulus signs: $||$ make a function positive.

$|x| = x$ for $x \geq 0$ and $|x| = -x$ for $x < 0$

Sequences and series

$n! = 1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \times n$

When $n > 1$ the following binomial expansions can be used:

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots + \binom{n}{r}x^r + x^n$$

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{r}a^{n-1}b^r + b^n$$

Trigonometry

Trigonometric Functions

secant x , $\sec x = 1 / (\cos x)$

cosecant x , $\operatorname{cosec} x = 1 / (\sin x)$

cotangent x , $\cot x = 1 / (\tan x)$

Simple Identities

$$\tan x \equiv (\sin x) / (\cos x)$$

$$\cot x \equiv (\cos x) / (\sin x)$$

$$\sin^2 x + \cos^2 x \equiv 1$$

$$\sec^2 x \equiv 1 + \tan^2 x$$

$$\operatorname{cosec}^2 x \equiv 1 + \cot^2 x$$

Compound Angle Formulae

$$\sin (A + B) \equiv \sin A \cos B + \cos A \sin B$$

$$\sin (A - B) \equiv \sin A \cos B - \cos A \sin B$$

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

$$\cos (A - B) \equiv \cos A \cos B + \sin A \sin B$$

$$\tan (A + B) \equiv (\tan A + \tan B) / (1 - \tan A \tan B)$$

$$\tan (A - B) \equiv (\tan A - \tan B) / (1 + \tan A \tan B)$$

Double Angle Formulae

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv 1 - 2 \sin^2 A$$

$$\cos 2A \equiv (2 \cos^2 A) - 1$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv (2 \tan A) / (1 - \tan^2 A)$$

Half-angle Formulae

$$\sin^2 (x/2) \equiv (1 - \cos x) / 2$$

$$\cos^2 (x/2) \equiv (1 + \cos x) / 2$$

Product → Sum Formulae

$$2 \sin A \cos B \equiv \sin (A + B) + \sin (A - B)$$

$$2 \cos A \sin B \equiv \sin (A + B) - \sin (A - B)$$

$$2 \cos A \cos B \equiv \cos (A + B) + \cos (A - B)$$

$$2 \sin A \sin B \equiv \cos (A - B) - \cos (A + B)$$

Sum → Product Formulae

$$\sin P + \sin Q = 2 \sin ((P + Q) / 2) \cos ((P - Q) / 2)$$

$$\sin P - \sin Q = 2 \cos ((P + Q) / 2) \sin ((P - Q) / 2)$$

$$\cos P + \cos Q = 2 \cos ((P + Q) / 2) \cos ((P - Q) / 2)$$

$$\cos Q - \cos P = 2 \sin ((P + Q) / 2) \sin ((P - Q) / 2)$$

Exponential and logarithms

Basic Concepts

$f(x) = a^x$ is an exponential function

$f(x) = e^x$ is *the* exponential function

$$\log_e x = \ln x$$

$$\log_{10} x = \lg x$$

If $f(x) = e^x$ then $f^{-1}(x) = \ln x$ ($x > 0$)

The laws of logarithms

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a (x / y) = \log_a x - \log_a y$$

$$\log_a x^n = n (\log_a x)$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_a b = (\log_c b) / (\log_c a)$$

Differentiation

$f(x)$	$f'(x)$
$k (e^x)$	$k (e^x)$
$\ln (k x)$	$1 / x$
$\log_a x$	$1 / (x \ln a)$

k is a constant, where appropriate.

Integration

function	Integral
$k e^x$	$k e^x + C$
$1 / x$	$\ln x + C$

Volume

The volume generated when a function, $f(x)$ is rotated 2π radians about the x -axis, the volume can be calculated using:

$$\pi \int_a^b [f(x)]^2 dx$$

Trapezium Rule

The area bound by the equation $y = f(x)$, the x -axis, and the ordinates a and b ($x = a$, $x = b$) can be approximated using:

$$0.5 h (y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + \dots + 2y_{n-2} + 2y_{n-1} + y_n)$$

Numerical methods

Iteration Formulae

To find an iteration formula, the equation $f(x)$ must first be rearranged to the form:

$$x = g(x)$$

The iteration formula, is then:

$$X_{n+1} = g(X_n)$$