P3

Algebra

The function:

$$\frac{f(x)}{(x-1)(x-2)(x-3)}$$

Has partial fractions:

$$\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

The function:

$$\frac{f(x)}{(x+2)(x^2+2x+3)}$$

Has partial fractions:

$$\frac{A}{x+2} + \frac{Bx+C}{X^2+2x+3}$$

The function:

$$\frac{f(x)}{(x+2)^3}$$

Has partial fractions:

$$\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$$

A fraction where the numerator is bigger than or equal to the denominator is referred to as an improper fraction.

The remainder theorem states that when a polynomial, f(x) is divided by (ax – b) the remainder is f(b/a)

If (x – b) is a factor, then f(b) = 0.

Differentiation

The chain rule

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

Product rule

When y = uv

$$\frac{dy}{dx} = v\frac{du}{dx} + u\,\frac{dv}{dx}$$

Quotient rule

When y = u/v

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\,\frac{dv}{dx}}{v^2}$$

Derivatives

f(x)	**f'(x)**
x^n	nx^{n-1}
$(ax + b)^n$	$na(ax + b)^{n-1}$
e^x	e^x
e^{ax+b}	ae^{ax+b}
a^x	a^x (ln a)
lnx	1 / x
ln(ax + b)	a / (ax + b)
sin x	cos x
cos x	-sin x
tan x	$\sec^2 x$
cot x	$-\text{cosec}^2 x$
sec x	sec x tan x
cosec x	-cosec x cot x
sin kx	k cos kx
$\sin^n x$	$n \sin^{n-1} x \cos x$
$\sin^n kx$	$nk \sin^{n-1}kx \cos kx$
y^n	$ny^{n-1} (^{dy}/_{dx})$
xy	$y + x (^{dy}/_{dx})$

If the equation of the curve is given in terms of parametric equations, in the form x = f(t) and y = g(t) then:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Integration

Standard results

f(x)	Integral of f(x)
sin x	-cos x + C
sin (ax + b)	(-1 / a) cos (ax + b) + C
cos x	sin x + C
cos (ax + b)	(1 / a) sin (ax + b) + C
$(ax + b)^n$ [n ≠ -1]	$(1 / (a(n + 1)))(ax + b)^{n+1}$ + C
1 / (ax + b)	(1 / a)(ln |ax + b|) + C
e^{ax+b}	$(1 / a)e^{ax+b}$
f(x) f'(x)	$0.5[f(x)]^2$
$[f(x)]^n$ f'(x) [n ≠ -1]	$(1 / n + 1)[f(x)]^{n+1}$ + C
f'(x) / f(x)	ln |f(x)| + C

Integration by parts

$$\int v \frac{du}{dx}\,dx = uv - \int u \frac{dv}{dx}\,dx$$

Coordinate geometry on the x-y plane

The circle, centre the origin and radius r has equation:

$x^2 + y^2 = r^2$

The circle, centre (a, b) and radius r has equation:

$(x - a)^2 + (y - b)^2 = r^2$

The circle, centre (-f, -g) with radius $\sqrt{(g^2 + f^2 - c)}$

$x^2 + y^2 + 2gx + 2fy + c = 0$

The tangent at (h, x) to the circle with equation $x^2 + y^2 + 2gx + 2fy + c = 0$ is:

$hx + ky + g(h + x) + f(k + y) + c = 0$

For the circle with centre (a, b) and radius r, suitable parametric equations are:

$x = a + r \cos x, \quad y = b + r \sin x, \quad 0 \le x < 2\pi$

Vectors

A unit vector is a vector whose |magnitude| = 1

The modulus of:

$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

$|a| = \sqrt{(x^2 + y^2 + z^2)}$

Scalar ("Dot") Product

Where:

$\mathbf{a} = x_1\mathbf{i} + x_1\mathbf{j} + x_1\mathbf{k}$

$\mathbf{b} = x_2\mathbf{i} + x_2\mathbf{j} + x_2\mathbf{k}$

$\mathbf{a.b} = x_1x_2 = y_1y_2 + z_1z_2 = |\mathbf{a}|\ |\mathbf{b}| \cos x$