Permutations and combinations

3 men and 5 women belong to a club. A committee of 4 people is to be formed. How many possible committees are there?

$$^{8}C_{4} = \frac{8!}{4! \cdot 4!} = 70$$

It is now decided that there should be an equal number of men and women on the committee.

$$^{3}C_{2} \times {}^{5}C_{2} = \frac{3!}{1! \cdot 2!} \cdot \frac{5!}{2! \cdot 3!} = 30$$

Combinations and selections, where the order does not matter can therefore be expressed as:

$$^{n}C_{r} = \frac{n!}{(n-r)! \cdot r!}$$

Permutations/arrangements (order does matter)

How many ways can you choose the order of the first 6 finishers in a race of 10 runners?

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 = \frac{10!}{(10-6)!} = \frac{10!}{4!}$$

So permutations and combinations, where the order does matter can therefore be expressed as:

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Alternatively, how many committees are there not all of the same gender?

All committees less those all all-female committees:

$$^{8}C_{4} - {}^{3}C_{0} \times {}^{5}C_{4} = 70 - 5 = 65$$

Or, all 2-2 committees, all 1-3 committees and all 3-1 committees:

$$^{3}C_{2} \times {}^{5}C_{2} + {}^{3}C_{1} \times {}^{5}C_{3} + {}^{3}C_{3} \times {}^{5}C_{1} = 30 + 30 + 5 = 65$$