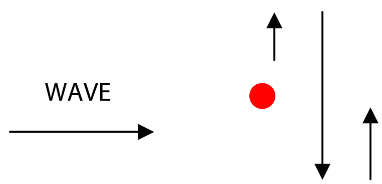
Physics Chapter 6: Waves

Waves transfer energy, and have a wavelength, such that $v = f\lambda$, where v is the velocity of the wave, f the frequency and λ the wavelength. Waves can be categorised into two categories:

Transverse

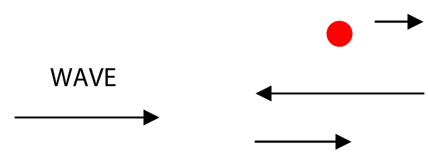
- Much more common (generally most waves are transverse).
- Particles in the medium oscillate perpendicular to direction of propogation of wave.



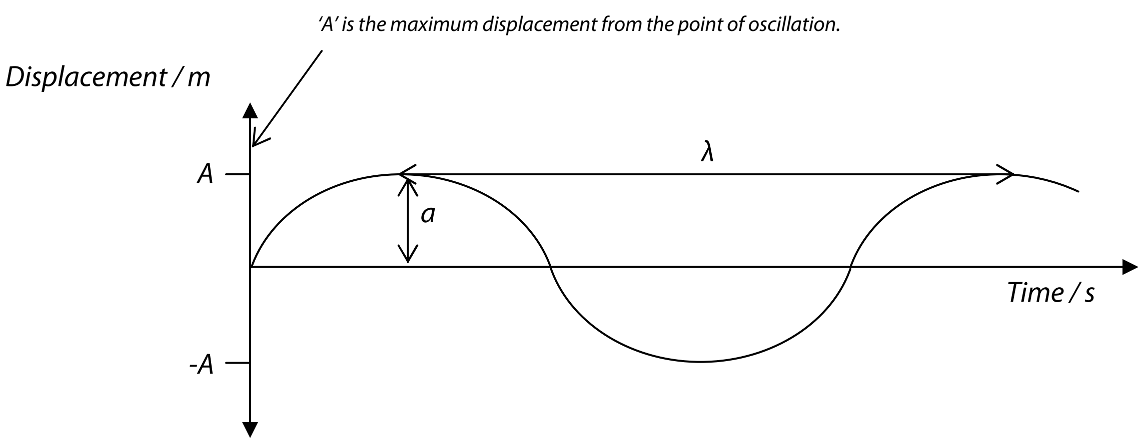
- The electromagnetic spectrum, waves in water and radio → γ are examples of transverse waves.

Longitudinal

- Much less common, sound and some earthquake waves are about the only examples.
- Particles in the medium oscillate parallel to direction of propogation of wave.



- Oscillation on a fixed point vibrate parallel to wave propogation.



f = number of oscillations per second, measured in Hz. It can be thought of as the number of waves that pass a point in one second.

T = the time period of one wave cycle or the time taken for one wave cycle to pass a point.

We can therefore say:

$$f = \frac{1}{T}$$

In one cycle the wave progresses a distance of *λ*, that is it moves *λ* metres. In *f* cycles it moves forward *fλ* metres, with *f* cycles produced per unit of time. This is how the equation $v = f\lambda$ is derived.

The oscillations of a wave can be modelled using a sinusoidal function, typically cosθ plotted against θ. When $t = T$, $\theta = 2\pi$ rad.

$$\frac{\theta}{t} = \frac{2\pi}{T} \qquad \text{so} \qquad \theta = \frac{2\pi t}{T}$$

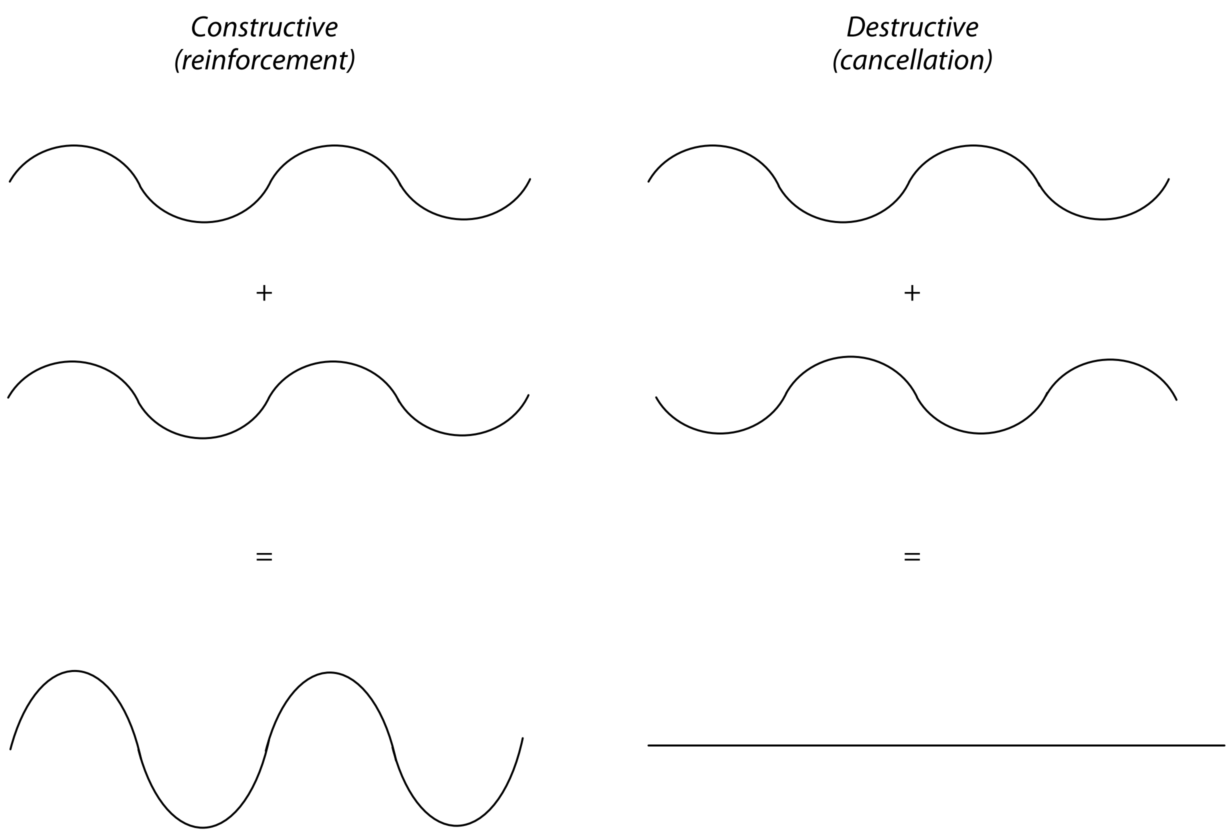
This is the angular velocity, ω, of the wave so $v = r\omega$ and $a = r\omega^2$ can be used here.

$$s = A\cos\omega t$$

Where s = displacement and A = maximum amplitude of the wave.

Superposition

The principal of superposition states that when waves meet their displacements can be added. Two important points of superposition are where constructive and destructive interference occurs.

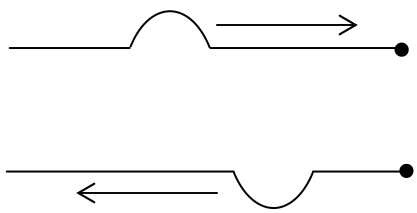


Wave reflection

$i = r$ for a flat surface.

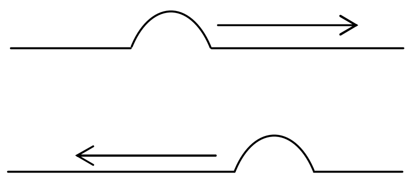
Phase changes:

Transverse pulse reflected from a fixed end



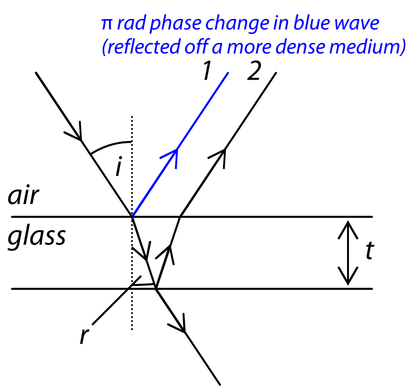
π rad inversion

Transverse pulse reflected from a free end

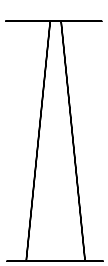


Reflection in same phase

Generally reflection at a denser or less responsive medium means waves are inverted π rad. For example, when light moves from air to glass it is refracted. The reflected ray has a π rad phase change, whereas the reflected ray from glass to air does not:



If the path difference is $2t\sec r$ (which for very small values of t, $2t\sec r \fallingdotseq 2t$), that is 2t is half a wavelength then waves 1 and 2 will be π rad out of phase. As the width of the film becomes thinner it approaches the wavelength of light (4-8nm), the distance between the two reflected waves becomes much smaller and so the two can superpose. If the thin film width is a quarter of a wavelength then the waves will destructively superpose.



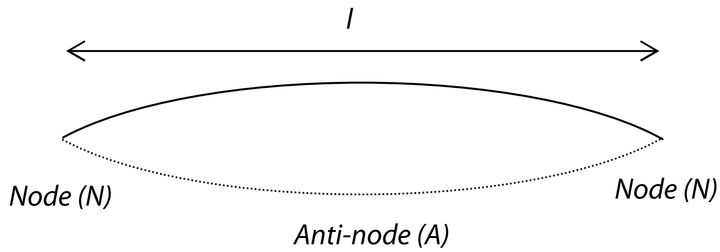
Film is thicker at the bottom that at the top due to gravity.

Horizontal bands of colour are observed as he phase difference has lead to superposition of a particular wavelength of light. This means that that wave length of light is not visible as it has been destructively superposed. The wavelength of this light is a multiple of $2t\sec r$, i.e. $2t\sec r$, $4t\sec r$, $6t\sec r \ldots 2nt\sec r$, where n is a positive integer.

The reason for these bands is when a film is held vertically gravity acts upon the film meaning it is not a uniform thickness; the film is thicker at the bottom than at the top. The bands appear to move because of convection currents. At the top at the thinnest part of the wave just before the film breaks may appear black or entirely transparent as $t \rightarrow 0$.

Standing waves

Standing waves are oscillations that store energy. Two waves of the same *A* and *f* move in opposite directions.

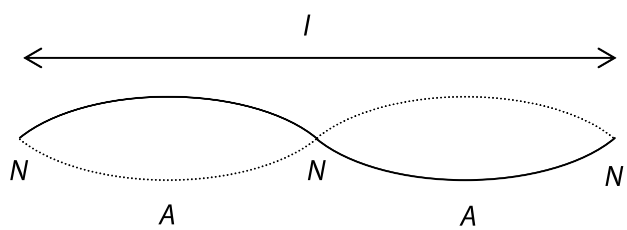


The first time a standing wave is formed is when $l = \frac{1}{2}\lambda$, which is called the first harmonic or fundamental resonance. Using the equation $v = f\lambda$, when the material and length are kept constant v must remain constant, so as f increases λ must decrease.

For the first harmonic:

$$l = \tfrac{1}{2}\lambda_1 \qquad \lambda_1 = 2l \qquad f_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$$

Another standing wave is formed when the frequency is increased such that:



This is called the second harmonic (notice that the *A* of the wave has decreased as the string has remained the same length). An entire wavelength is seen in the length of the string, so here:

$$l = \lambda_2 \qquad f_2 = \frac{v}{\lambda_2} = \frac{v}{l} = 2f_1$$

And for the third harmonic:

$$l = 1.5 * \lambda_3 \qquad f_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3f_1$$

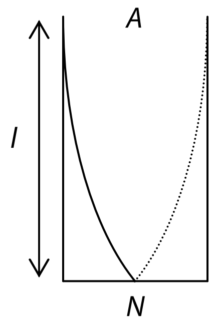
So, for the nth harmonic:

$$f_n = n * f_1$$

Standing waves in air columns

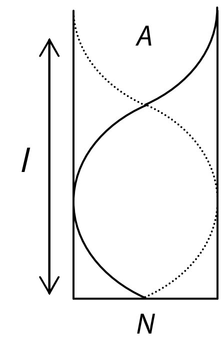
Resonance occurs when one wavelength fits into the space in the bottle, for example when you blow across the top of a milk bottle. If you fill the bottle with water a higher note is produced. This is because v has remained constant λ has decreased so f has had to increase.

Air column with one closed end and one open end:



Maximum longitudinal displacement of particles at *A*, the anti-node, as particles are free to move in and out of the open end of the air column. The layer of the air closest to the bottom cannot move so this is a displacement node. This is the first harmonic or the fundamental harmonic.

Another standing wave is created when the frequency increases three times to $3f_1$. This is called the first overtone or the second harmonic. Here the wavelength, λ_2 is a sixth of λ_1 as shown on the right.



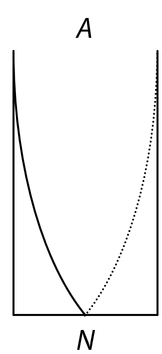
Pressure

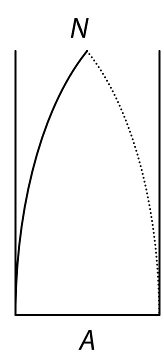
Alternatively standing waves in pipes can be represented by pressure.

- At the open end there is a constant atmospheric pressure; a pressure node.
- At the closed end pressure will vary (as the particles are oscillating most); a pressure anti-node.

Where there is maximum motion of air particles (a displacement anti-node) there is minimum pressure variation (a pressure node).

So: *Displacement:* *Pressure:*





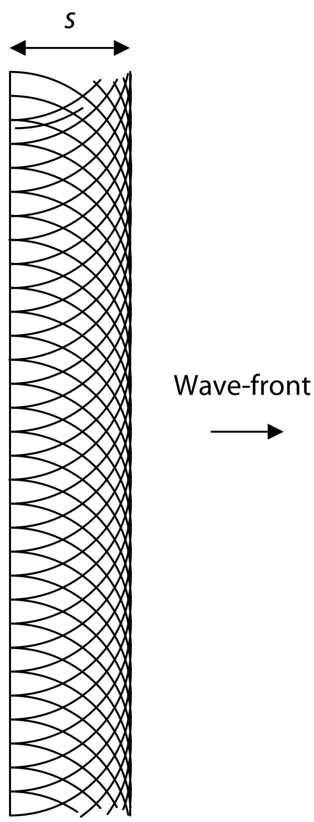
Light as a wave

Light is part of the electromagnetic spectrum. It therefore has and transfers energy. It is a transverse wave and travels at a speed ~$3 * 10^8 ms^{-1}$, often denoted by the letter *c*.

The argument for light being a particle – a photon – is discussed in chapter 7 using ideas such as $E = hf = mc^2$, where *m* is the mass equivalent and *E* the energy.

Huygen's construction

Huygen's construction predicts the position of a wave-front at a point in the future after a particular event using the concept on secondary wavelets. This is that all points on an original wave-front act as a source of wavelets.



$$s = vt$$

when t = T, the time period for one wave

$$s = vT$$

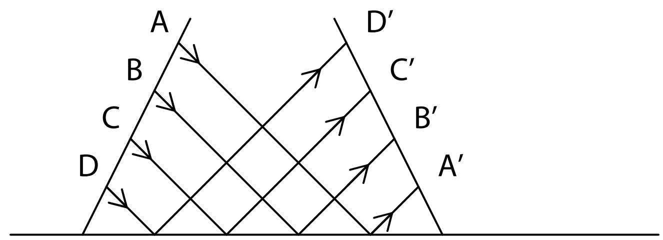
but v = fλ implies

$$s = f\lambda T$$

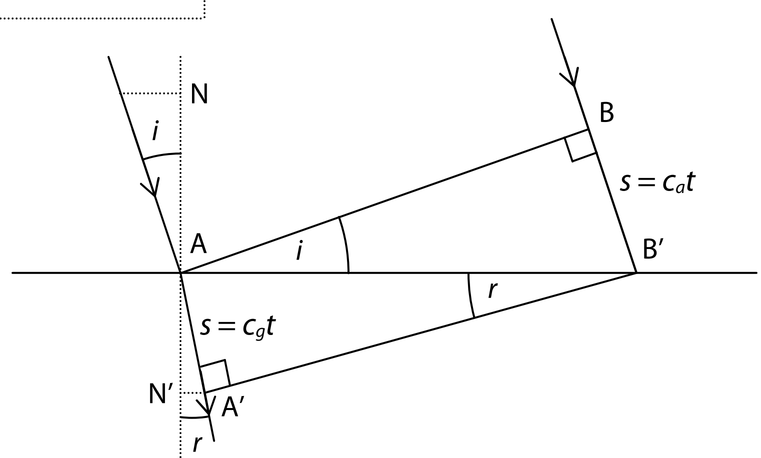
but $f = T^{-1}$ implies

$$s = \lambda$$

Huygen's construction can also be used to demonstrate reflection of waves off a flat surface from ABCD to A′B′C′D′.



The concept can also be used to explain refraction and from this derive Snell's law.



In triangle BAB′ and A′B′A:

$$\frac{\sin i}{\sin r} = \frac{\sin BAB'}{\sin A'B'A} = \frac{BB' / AB'}{AA' / AB'} = \frac{BB'}{AA'} = \frac{c_a t}{c_g t} = \frac{c_a}{c_g} \approx \frac{3 * 10^8}{2 * 10^8} = 1.5$$

Which is Snell's law, that the ratio of velocities in air and glass is ${}_a n_g \approx 1.5$.

Experiments with light

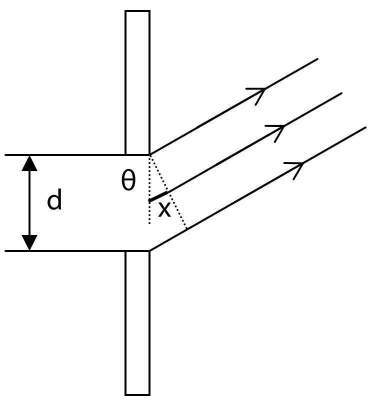
Coherent sources of waves are sources which maintain a constant phase difference.

- Energy is emitted as electrons drop in energy levels.
- Only for a few nanoseconds (10^{-9} seconds).
- Emit a train 0.3m long (half a million cycles).
- Not constant cycle as atoms bump into each other. There are sections of perfect cycles then abrupt changes.

To create a coherent waves source wave fronts can be divided, through single slit wave diffraction or the amplitude can be divided using a thin film for example.

The wavelength of visible light ranges from blue, 400nm to red, 700nm. Here *d* represents the aperture, the width of the gap.

For the first minimum a ray near one edge of the slit cancels with one at the centre. The path difference between the two rays must be half a wavelength for the first minimum.

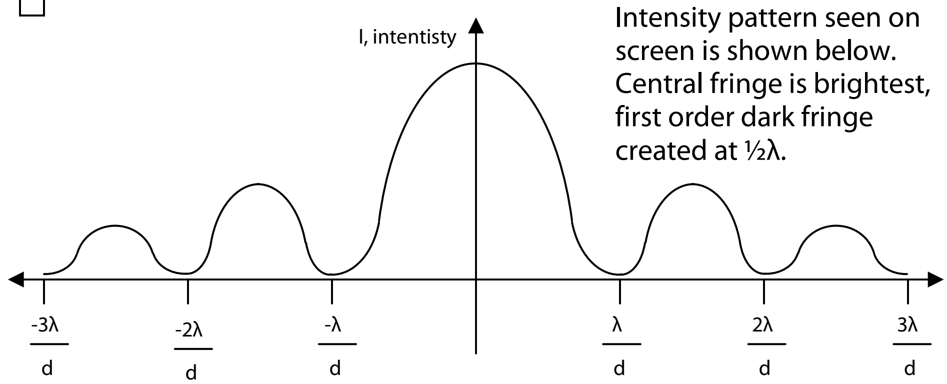


When *d* is small the wave is diffracted through the aperture. Path difference of x to interfere destructively needs to be ½λ. For x = ½λ, sinθ ≒ θ so θ ≒ ½λ.

The nth minimum occurs at:

$$\sin\theta_n = \frac{n\lambda}{d} \quad \leftrightarrow \quad n\lambda = d\sin\theta_n$$

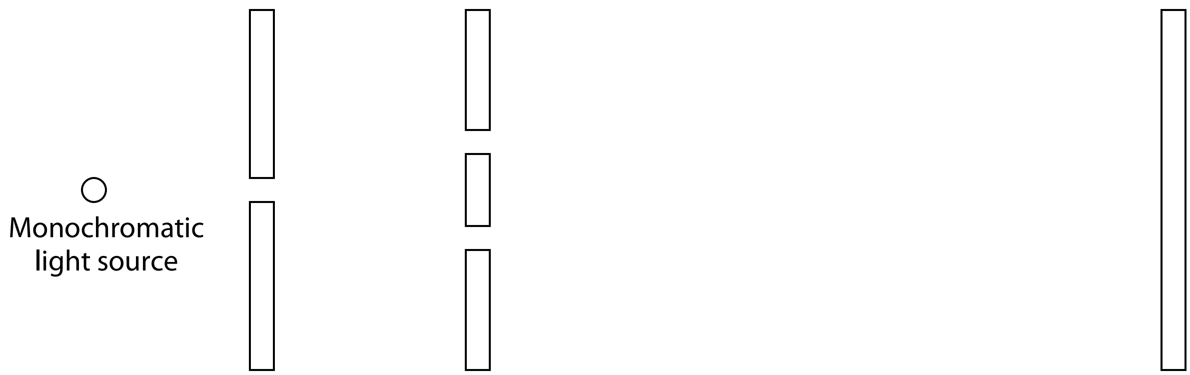
Intensity pattern seen on screen is shown below. Central fringe is brightest, first order dark fringe created at ½λ.



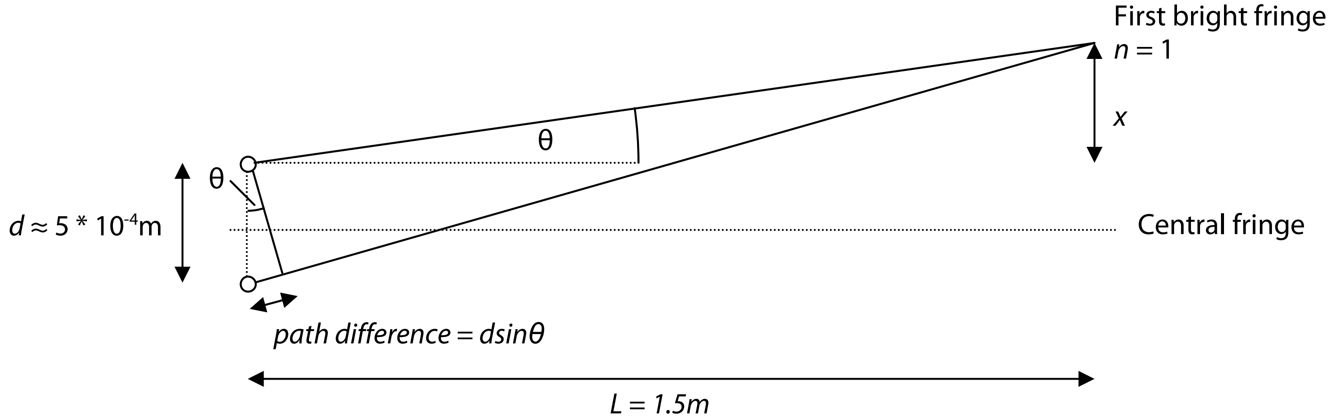
Young's double slit experiment

Main principles: superposition and interference, creation of regular pattern of interference fringes.

Light waves are intermittent quantised jumps of electrons in individual atoms – they are incoherent. White light contains a spectrum of main different wavelengths – it is polychromatic. A laser has just one wavelength, it is monochromatic. For this reason lasers demonstrate Young's double slit experiment well.



Light diffracts through a narrow single slit and coherence is achieved – that is the waves have a constant phase difference. This means that superposition occurs and a stable interference pattern is created.



Note that here *d* means the space between the two slits, not the width of the slit as in the previous single slit experiment.

$$\sin\theta \fallingdotseq \frac{x}{L} \qquad \text{(as } \sin\theta \fallingdotseq \tan\theta \text{ for small } \theta\text{)}$$

$$x \fallingdotseq \frac{\lambda L}{d}$$

A diffraction grating is lots of very fine slits together. The number of slits is expressed in lines/mm, typically 300+.

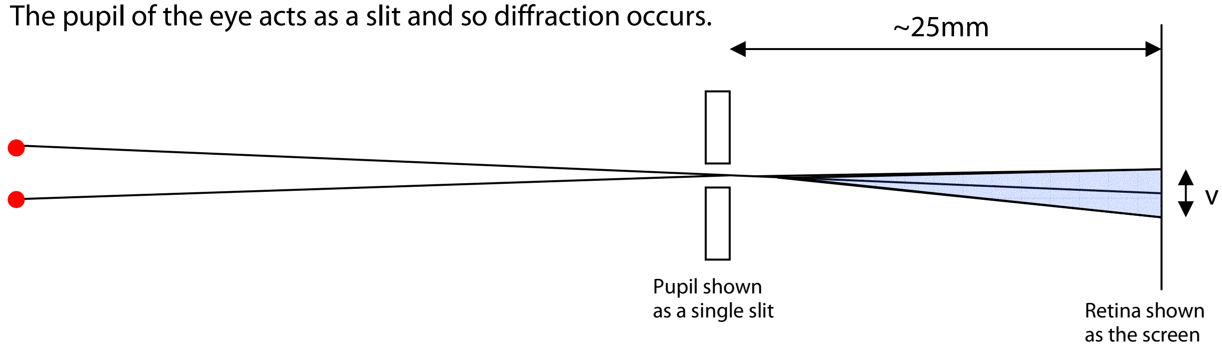
If white light is used fewer distinct fringes are seen as each colour produces its own set of fringes, and these overlap. Red light diffracts more (as the value of λ is larger). To make blue fringes more visible d could be decreased or L could be increased. This will lead to x increasing.

The number of slits visible depends on the number of fringes, and the theory suggests some fringes would be created that are not visible (i.e. where θ is bigger than 90°). d/λ (truncated) gives the number of fringes that are within 90°.

Diffraction at a single slit

This explains why in the distance at night we cannot make out two distinct beams from car headlights and how most printing technologies work – using small dots of ink.

The pupil of the eye acts as a slit and so diffraction occurs.



If the beam from the lower image falls within the region *v* then the two dots cannot be resolved by the eye. If the beam falls outside this region then the eye can resolve the two dots. The blue region shows where the two dots could not be resolved.