Modelling radioactive decay

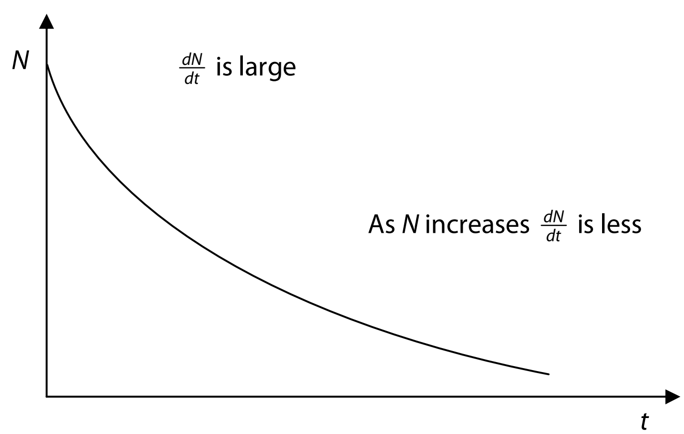
An unstable nucleus has a certain probability of decay. We cannot predict exactly when decay will happen as it is a random process.

Let the probability of decay of a nucleus in one second, the standard unit of time, be λ.

If the number of nuclei is initially *N*, and after one second the change in *N*, ΔN, is given by $\Delta N = -N\lambda$, and after a time Δt this is $\Delta N = -N\lambda\Delta t$ (for small Δt). Note the negative sign, this is because as time *t* passes the number of nuclei that have not decayed gets smaller, meaning the change in *N* is negative.

Rearranging $\Delta N = -N\lambda\Delta t$ gives $\frac{\Delta N}{\Delta t} = -N\lambda$, or $\frac{dN}{dt} = -N\lambda$ considering calculus*. $\frac{dN}{dt}$ therefore represents the rate of activity. In practise if this were to be measured all decaying particles would need to be collected.

Let N_0 be the initial number of nuclei that have not decayed. We see the following graph.



Mathematically we can solve the differential equation $\frac{dN}{dt} = -N\lambda$ to find an equation for this line (requires understanding of P3 integration by separable variables).

$$\frac{dN}{dt} = -N\lambda$$

$$\int_{N_0}^{N} \tfrac{1}{N}\,dN = \int_0^t -\lambda\,dt$$

$$\left[\ln N\right]_{N_0}^{N} = \left[-\lambda t\right]_0^t$$

$$\ln\left(\tfrac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

* *dN* can be thought of as a mathematical refinement of ΔN, a small change in *N*.

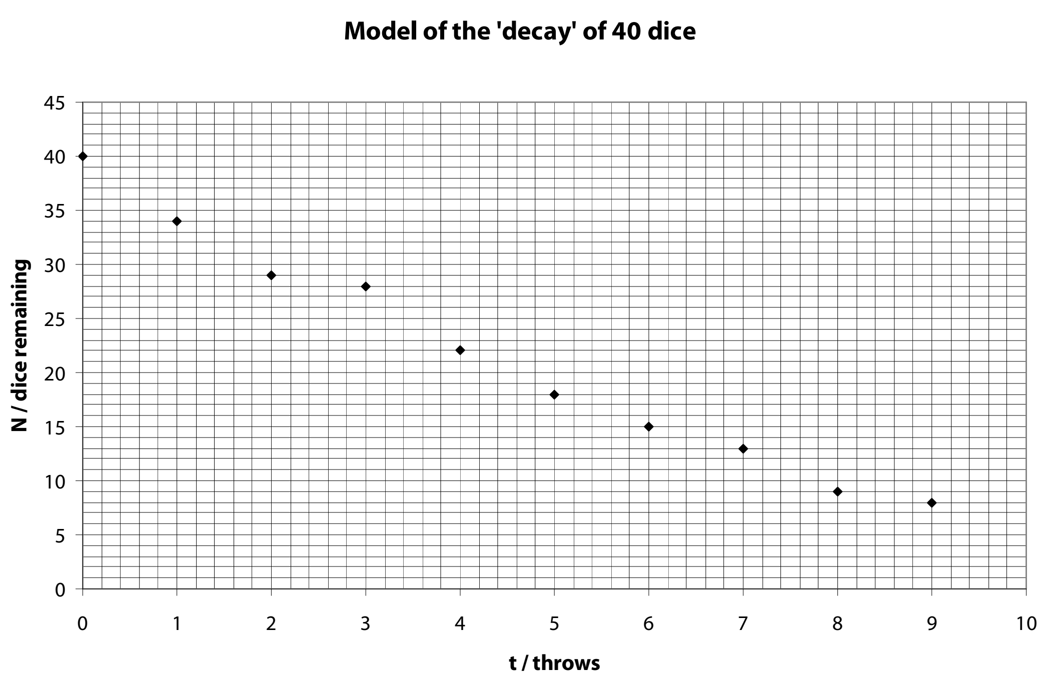
Whilst most often the final form is the most useful it is also worth remembering $\ln\left(\frac{N}{N_0}\right) = -\lambda t$ for instances when you are required to find *t*.

Modelling decay

Using 40 dice model decay. If the dice throws a 6, it acts as having decayed. The following results were obtained.

***t* / throws**	***N* / dice remaining**
0	40
1	34
2	29
3	28
4	22
5	18
6	15
7	13
8	9
9	8

These results plot to form the following graph, on which the proposed relationship above can be approximately seen.



These notes are from a lesson on 01/12/2004.